order 3 we obtain five magic parallelopipeds of order $3 \times 3 \times 5$ together forming an associated magic octahedroid of order $3 \times 3 \times 5 \times 5$. Since the lengths of the edges are the same as those of the octahedroid formed from Fig. 7 square, these two four-dimensional figures are identical but the distribution of the numbers in their cells is not the same. They can however be made completely identical both in form and distribution of numbers by a slight change in our method of dealing with the square Fig. 6, i. e., by taking the square plates to form the parallelopipeds from the knight paths instead of the diagonals. Using the path $-1, 2$ we get 225, 106, 3, 188, 43 for the first plates of each parallelopiped, and then using $2, -1$ for the successive plates of each, we obtain the parallelopipeds:

I.	225,	8,	31,	118,	183
II.	106,	193,	213,	15,	38
III.	3,	45,	113,	181,	223
IV.	188,	211,	13,	33,	120
V.	43,	108,	195,	218,	1

This octahedroid is completely identical with that previously obtained from Fig. 7, as can be easily verified by taking any number at random and writing down the four series of numbers through its containing cell parallel to the edges, first in one octahedroid and then in the other. The sets so obtained will be found identical.

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PANDIAGONAL-CONCENTRIC MAGIC SQUARES OF ORDERS 4*m*.

These squares are composed of a central pandiagonal square surrounded by one or more bands of numbers, each band, together with its enclosed numbers, forming a pandiagonal magic square.

The squares described here are of orders 4*m* and the bands or borders are composed of double strings of numbers. The central square and bands are constructed simultaneously instead of by the usual method of first forming the nucleus square and arranging the bands successively around it.

[2] *The Theory of Path Nasiks*, by C. Planck, M.A., M.R.C.S., printed by A. J. Lawrence, Rugby, Eng.

A square of the 8th order is shown in Fig. 1, both the central 4^2 and 8^2 being pandiagonal. It is 4^2 ply, i. e., any square group of 16 numbers gives a constant total of $8(n^2+1)$, where n = the number of cells on the edge of the magic. It is also magic in all of its Franklin diagonals; i. e., each diagonal string of numbers bending at right angles on either of the horizontal or vertical center lines of the square, as is shown by dotted lines, gives constant totals. In any size concentric square of the type here described, all of its concentric squares of orders $8m$ will be found to possess the Franklin bent diagonals.

The analysis of these pandiagonal-concentric squares is best illustrated by their La Hirean method of construction, which is

45	28	35	22	47	26	33	24
49	8	63	10	51	6	61	12
31	42	17	40	29	44	19	38
3	54	13	60	1	56	15	58
46	27	36	21	48	25	34	23
50	7	64	9	52	5	62	11
32	41	18	39	30	43	20	37
4	53	14	59	2	55	16	57

Fig. 1.

here explained in connection with the 12th order square. The square lattice of the subsidiary square, Fig. 2, is, for convenience of construction, divided into square sections of 16 cells each. In each of the corner sections (regardless of the size of the square to be formed) are placed four 1's, their position to be as shown in Fig. 2. Each of these 1's is the initial number of the series 1, 2, 3,..... $(n/4)^2$, which must be written in the lattice in natural order, each number falling in the same respective cell of a 16-cell section as the initial number. Two of these series are indicated in Fig. 2 by circles enclosing the numbers, and inspection will show that each of the remaining series of numbers is written in the lattice in the same manner, though they are in a reversed or reflected order. Any size subsidiary square thus filled possesses all the magic features of the final square.

A second subsidiary square of the 4th order is constructed with the series 0, $(n/4)^2$, $2(n/4)^2$, $3(n/4)^2$,......$15(n/4)^2$, which must be so arranged as to produce a pandiagonal magic such as is shown in Fig. 3. It is obvious that if this square is pandiagonal, several of these squares may be contiguously arranged to form a larger

1	9	7	3	4	6	4	6	7	3	1	9
1	9	7	3	4	6	4	6	7	3	1	9
7	3	1	9	4	6	4	6	1	9	7	3
7	3	1	9	4	6	4	6	1	9	7	3
2	8	8	2	5	5	5	5	8	2	2	8
2	8	8	2	5	5	5	5	8	2	2	8
8	2	2	8	5	5	5	5	2	8	8	2
8	2	2	8	5	5	5	5	2	8	8	2
3	7	9	1	6	4	6	4	9	1	3	7
3	7	9	1	6	4	6	4	9	1	3	7
9	1	3	7	6	4	6	4	3	7	9	1
9	1	3	7	6	4	6	4	3	7	9	1

Fig. 2.

99	54	72	45
108	9	135	18
63	90	36	81
0	117	27	126

Fig. 3.

square that is pandiagonal and 4^2-ply, and also has the concentric features previously mentioned.

Fig. 3 is now added to each section of Fig. 2, cell to cell, which will produce the final magic square in Fig. 4.

With a little practice, any size square of order $4m$ may be con-

structed without the use of subsidiary squares, by writing the numbers directly into the square and following the same order of numeral procession as shown in Fig. 5. Other processes of direct construction may be discovered by numerous arrangements and combinations of the subsidiary squares.

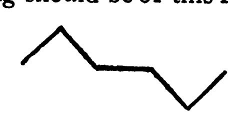
Fig. 5 contains pandiagonal squares of the 4th, 8th, 12th and 16th orders and is 4^2-ply. The 8th and 16th order squares are also magic in their Franklin bent diagonals.

These concentric squares involve another magic feature in

100	63	79	48	103	60	76	51	106	57	73	54
109	18	142	21	112	15	139	24	115	12	136	27
70	93	37	90	67	96	40	87	64	99	43	84
7	120	28	135	4	123	31	132	1	126	34	129
101	62	80	47	104	59	77	50	107	56	74	53
110	17	143	20	113	14	140	23	116	11	137	26
71	92	38	89	68	95	41	86	65	98	44	83
8	119	29	134	5	122	32	131	2	125	35	128
102	61	81	46	105	58	78	49	108	55	75	52
111	16	144	19	114	13	141	22	117	10	138	25
72	91	39	88	69	94	42	85	66	97	45	82
9	118	30	133	6	121	33	130	3	124	36	127

Fig. 4.

respect to zig-zag strings of numbers. These strings pass from side to side, or from top to bottom, and bend at right angles after every fourth cell as indicated by the dotted line in Fig. 5. It should be noted, however, that in squares of orders $8m+4$ the central four numbers of a zig-zag string must run parallel to the side of the square, and the string must be symmetrical in respect to the center line of the square which divides the string in halves. For example in a square of the 20th order, the zig-zag string should be of this form

 and not of this form 

In fact any group or string of numbers in these squares, that is symmetrical to the horizontal or vertical center line of the magic and is selected in accordance with the magic properties of the 16-cell subsidiary square, will give the sum $[r(n^2+1)]/2$, where r = the number of cells in the group or string, and n = the number of cells in the edge of the magic. One of these strings is exemplified in Fig. 5 by the numbers enclosed in circles.

1	224	61	228	5	220	57	232	9	216	53	236	13	212	49	240
113	176	77	148	117	172	73	152	121	168	69	156	125	164	65	160
205	20	241	48	201	24	245	44	197	28	249	40	193	32	253	36
189	100	129	96	185	104	133	92	181	108	137	88	177	112	141	84
2	223	62	227	6	219	58	231	10	215	54	235	14	211	50	239
114	175	78	147	118	171	74	151	122	167	70	155	126	163	66	159
206	19	242	47	202	23	246	43	198	27	250	39	194	31	254	35
190	99	130	95	186	103	134	91	182	107	138	87	178	111	142	83
3	222	63	226	7	218	59	230	11	214	55	234	15	210	51	238
115	174	79	146	119	170	75	150	123	166	71	154	127	162	67	158
207	18	243	46	203	22	247	42	199	26	251	38	195	30	255	34
191	98	131	94	187	102	135	90	183	106	139	86	179	110	143	82
4	221	64	225	8	217	60	229	12	213	56	233	16	209	52	237
116	173	80	145	120	169	76	149	124	165	72	153	128	161	68	157
208	17	244	45	204	21	248	41	200	25	252	37	196	29	256	33
192	97	132	93	188	101	136	89	184	105	140	85	180	109	144	81

Fig. 5.

To explain what is meant above in reference to selecting the numbers in accordance with the magic properties of the 16-cell subsidiary square, note that the numbers, 27, 107, 214, 166, in the exemplified string, form a magic row in the small subsidiary square, 70, 235, 179, 30 and 251, 86, 14, 163 form magic diagonals, and 66, 159, 255, 34 and 141, 239, 82, 52 form ply groups.

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